

Phenomenology of the little flavon model

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Abstract

The phenomenology of the little-flavon model is discussed. Flavor changing neutral current and lepto-quark compositeness set the most stringent bounds to the lowest possible value for the scale at which the flavons arise as pseudo-Goldstone bosons.

I. MOTIVATIONS

The little-flavon model [1] explains fermion masses and mixing matrices in a little-Higgs inspired scenario [2]. In the model, an $SU(2)_F \times U(1)_F$ gauge flavor symmetry is spontaneously and completely broken by the vacuum of the dynamically induced potential for two scalar doublets, the little-flavons, which are pseudo-Goldstone bosons remaining after the spontaneous breaking—at a scale $\Lambda = 4\pi f$ —of an approximate $SU(6)$ global symmetry. The $SU(2)_F \times U(1)_F$ flavor symmetry is the diagonal combinations of a $[SU(2) \times U(1)]^2$ gauge symmetry surviving the spontaneous breaking of $SU(6)$.

The model reproduces successfully fermion masses and mixing angles [1]. Ratios between the values of the vacuum expectation values of the flavons and f give rise to the textures in the fermion mass matrices. For this reason fermion masses and mixing angles predicted by the model are independent of the scale f . On the other hand, the masses of the flavor bosons—both vector and scalar—arising in the breaking of the gauge symmetries depend on f . Since these masses enter in the processes mediated by the new particles, a computation of their effects allows us to constrain possible values of f .

In this letter, several processes are considered together with their experimental bounds. We present the relevant processes that occur at tree level starting from those that give the less stringent bounds on f to that that give the most stringent. The latter bound comes from flavor changing neutral current in the $K^0-\bar{K}^0$ for which we have $\Lambda \geq 5 \times 10^4$ TeV. We also give an example of a one-loop process that gives a limit on f as stringent as that coming from $K^0-\bar{K}^0$. Finally we show how the electroweak Higgs mass scale ($\simeq 200$ GeV) can be kept stable notwithstanding the much higher scale of flavor physics.

II. INTERACTIONS

The effective lagrangian at the scale Λ is given by

$$\mathcal{L} = \mathcal{L}_{kin}^\Sigma + \mathcal{L}_{kin}^f + \mathcal{L}_{kin}^g + \mathcal{L}_Y, \quad (1)$$

where $\mathcal{L}_{kin}^{\Sigma,f,g}$ includes the kinetic terms for the pseudo-Goldstone bosons, the fermions and the gauge bosons respectively and \mathcal{L}_Y the Yukawa couplings. Explicitly we have

$$\mathcal{L}_{kin}^\Sigma = -\frac{f^2}{4} \text{Tr} (D^\mu \Sigma) (D_\mu \Sigma)^*,$$

$$\mathcal{L}_{kin}^f = \bar{f}_{L,R} \gamma_\mu (\partial^\mu + i g_1 A_{1a}^\mu T^a + i g'_1 B_1^\mu) f_{L,R} , \quad (2)$$

with

$$D^\mu \Sigma = \partial^\mu + i g_i A_{ia}^\mu (Q_i^a \Sigma + \Sigma Q_i^{aT}) + i g'_i B_i^\mu (Y_i \Sigma + \Sigma Y_i^T) , \quad (3)$$

where $\Sigma = \exp(i\Pi/f)\Sigma_0$ is the non linear representations of the pseudo-Goldstone bosons, A_{ia}^μ , B_i^μ , ($i = 1, 2$; $a = 1, 2, 3$), the gauge bosons of the two copies of $SU(2) \times U(1)$ and g_i , g'_i their couplings. \mathcal{L}_Y is rather cumbersome and we report its expression in the Appendix.

After the spontaneous breaking of global $SU(6)$ we are left with four massive gauge bosons, $A_a^{\prime\prime\mu}$ ($a = 1, 2, 3$) and $B^{\prime\prime\mu}$ of masses

$$m_{A'}^2 = \frac{(g_1^2 + g_2^2)}{2} f^2 \quad \text{and} \quad m_{B'}^2 = \frac{2}{5} (g_1'^2 + g_2'^2) f^2 , \quad (4)$$

and four massless gauge bosons, A_a^μ ($a = 1, 2, 3$) and B^μ .

After the $SU(2)_F \times U(1)_F$ symmetry is broken we are left with one complex massive gauge boson, F_3^μ , 2 real massive gauge bosons, $F_{1,2}^\mu$, 2 real, $\varphi_{1,2}$, and one complex, φ_3 , massive scalars, which are the flavon scalars. Their masses are given by

$$\begin{aligned} m_{F_3}^2 &= \frac{1}{2} g^2 (\epsilon_1^2 + \epsilon_2^2) f^2 , \\ m_{F_{1,2}}^2 &= \frac{1}{2} (g^2 + g'^2) \epsilon_{1,2}^2 f^2 , \\ m_{\varphi_{1,2}}^2 &= [(\lambda_1 \epsilon_1^2 + \lambda_2 \epsilon_2^2 \pm \\ &\quad \sqrt{(\lambda_1 \epsilon_1^2 + \lambda_2 \epsilon_2^2)^2 - (4\lambda_1 \lambda_2 - \lambda_3^2) \epsilon_1^2 \epsilon_2^2}) f^2] , \\ m_{\varphi_3}^2 &= \frac{1}{2} \lambda_4 (\epsilon_1^2 + \epsilon_2^2) f^2 , \end{aligned} \quad (5)$$

where $g^2 = g_1^2 g_2^2 / (g_1^2 + g_2^2)$, $g'^2 = g_1'^2 g_2'^2 / (g_1'^2 + g_2'^2)$ are the effective gauge couplings, $\lambda_4 \simeq \mathcal{O}(1)$ and $\lambda_{1,2,3} \simeq \mathcal{O}(10^{-2})$ are the parameters of the potential as discussed in [1] and $\epsilon_{1,2}$ the ratios of the vacuum expectation values of ϕ_1 and ϕ_2 and the scale f . The numerical analysis in [1] indicates that $\epsilon_1 \epsilon_2 \simeq 0.2$ if we want to fit the fermion masses and mixing angles. The gauge bosons $F_{1,2}^\mu$ come from the mixing between A_3^μ and B^μ , while F_3^μ from the mixing between A_1^μ and A_2^μ .

From the kinetic term in eq. (2) we have the following interactions between the gauge bosons and the fermions

$$y_{F_{L,R}}^f \left(\frac{\sqrt{g^2 + g'^2}}{\sqrt{2}} \right) (\bar{f}_{L,R} \gamma_\mu f_{L,R}) (F_1^\mu + F_2^\mu) , \quad (6)$$

if $f_{L,R}$ is a singlet of $SU(2)_F$ with flavor hypercharge $y_{F_{L,R}}^f$, and

$$\frac{g}{\sqrt{2}} \left[(\bar{\psi}_{L,R}^1 \gamma_\mu \psi_{L,R}^2) F_3^{\dagger\mu} + h.c. \right] + \frac{\sqrt{g^2 + g'^2}}{\sqrt{2}} \left[(\bar{\psi}_{L,R}^1 \gamma_\mu \psi_{L,R}^1) F_2^\mu + (\bar{\psi}_{L,R}^2 \gamma_\mu \psi_{L,R}^2) F_1^\mu \right], \quad (7)$$

if $\psi_{L,R}$ is a doublet of $SU(2)_F$ of flavor hypercharge 1/2 with components $\psi_{L,R}^1$ and $\psi_{L,R}^2$.

The interactions in eqs. (6)–(7) have been written in the flavor current basis for the fermions $f_{L,R}$ and $\psi_{L,R}$. In the next sections we will indicate as $e_{L,R}^{i=1,2,3}$ the charged lepton flavor current eigenstates, $e_{L,R}^{\alpha=1,2,3} = e_{L,R}, \mu_{L,R}, \tau_{L,R}$ the charged lepton mass eigenstates, $L_{i,\alpha}^e, R_{i,\alpha}^e$ the unitary matrices that diagonalize the non diagonal mass matrix M_e^{RL} through the bi-unitary transformation

$$R^{e\dagger} M_e^{RL} L^e = M_e^{RL^{diag}}. \quad (8)$$

The same conventions will be used for the quarks, where we have $u_{L,R}^i, d_{L,R}^i, u_{L,R}^{\alpha=1,2,3} = u_{L,R}, c_{L,R}, t_{L,R}$ and $d_{L,R}^{\alpha=1,2,3} = d_{L,R}, b_{L,R}, s_{L,R}$, $L_{i,\alpha}^{u,d}, R_{i,\alpha}^{u,d}$ and M_u^{RL} and M_d^{RL} . For completeness all the non diagonal mass matrices are reported in the Appendix.

In the model all the Standard Model quarks are $SU(2)_F$ singlets charged under $U(1)_F$. Standard Model leptons belonging to the first family, l_{eL} and e_R , are $SU(2)_F$ singlets as well, while those of the second and third family, $l_{\mu,\tau L}$ and $(\mu, \tau)_R$, are members of a doublet in flavor space (see Tab. (I) in the Appendix). A consequence of this choice is that all lepton mass eigenstates interact with all the gauge bosons after $SU(2)_F \times U(1)_F$ is completely broken. For this reason it is useful to write down the interactions between flavor gauge bosons and charged lepton mass eigenstates. The general interaction is given by

$$y_{m_{L,R}}^{e\alpha\beta} \left(\frac{\sqrt{g^2 + g'^2}}{\sqrt{2}} \right) (\bar{e}_{L,R}^\alpha \gamma_\mu e_{L,R}^\beta) F_m^\mu, \quad (9)$$

where $m = 1, 2, 3$ and $y_{m_{L,R}}^{\alpha\beta}$ are given by

$$\begin{aligned} y_{1_U}^{e\alpha\beta} &= U_{1\alpha}^{e*} U_{1\beta}^e y_U^{e1} + U_{3\alpha}^{e*} U_{3\beta}^e, \\ y_{2_U}^{e\alpha\beta} &= U_{1\alpha}^{e*} U_{1\beta}^e y_U^{e1} + U_{2\alpha}^{e*} U_{2\beta}^e, \\ y_{3_U}^{e\alpha\beta} &= \sqrt{\frac{g^2}{g^2 + g'^2}} U_{2\alpha}^{e*} U_{3\beta}^e, \end{aligned} \quad (10)$$

with $U^e = L^e, R^e$ and y_U^{e1} the first family charged leptons flavor hypercharges (see Tab. (I) in the Appendix).

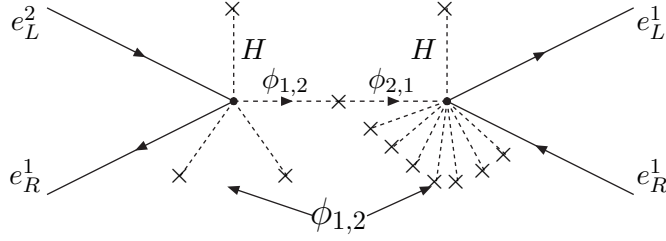


FIG. 1: Flavon mediated contribution to the decay $\mu^- \rightarrow e^+ e^+ e^-$. The fields $\phi_{1,2}$ are the $SU(2)_F$ doublets.

For completeness we report also the interaction between flavor gauge bosons and quark mass eigenstates. The interaction is given by

$$y_{L,R}^{q\alpha\beta} \left(\frac{\sqrt{g^2 + g'^2}}{\sqrt{2}} \right) (\bar{q}_{L,R}^\alpha \gamma_\mu q_{L,R}^\beta) (F_1^\mu + F_2^\mu), \quad (11)$$

where $y_{L,R}^{q\alpha\beta}$ are given by

$$y_U^{q\alpha\beta} = \sum_{i=1,2,3} U_{i\alpha}^{q*} U_{i\beta}^q y_U^{qi}, \quad (12)$$

with $U^q = L^q, R^q$ and y_U^{qi} the quarks flavor hypercharges (see Tab. (I) in the Appendix).

III. PROCESSES MEDIATED BY THE FLAVONS

All interactions between fermions and flavons come from the Yukawa lagrangian \mathcal{L}_Y in eq. (1) (see the Appendix for the full expression). These are the terms that give origin to the fermion mass matrices. After the breaking of $SU(2)_F \times U(1)_F$ it gives also the interactions we are interested here. Notice that in the following, for simplicity, we will indicate as *flavons* both the $SU(2)_F$ doublets, $\phi_{1,2}$, and the massive scalars, $\varphi_{1,2,3}$, arising after the breaking of the $SU(2)_F \times U(1)_F$ symmetry. Processes mediated by the flavons can occur at tree level and at one or more loops. Tree level processes concern direct interactions between fermions and only one flavon, for this reason couplings of this kind will follow the fermion mass matrices and all the flavor changing processes mediated by the flavons will be very suppressed since they will result be proportional to power of the ratio between the light fermion masses and the scale f .

In trying to constraint the flavon masses, let us first consider the lepton flavor violation (LFV) process $\mu \rightarrow 3e$. The limit on the branching ratio $\Gamma_{\mu^- \rightarrow e^+ e^+ e^-}$ is given as a function

of the total branching ratio $\Gamma_{\mu \rightarrow \text{all}}$ [3]

$$\frac{\Gamma_{\mu \rightarrow 3e}}{\Gamma_{\mu \rightarrow \text{all}}} < 10^{-12}, \quad (13)$$

with

$$\Gamma_{\mu \rightarrow \text{all}} = \frac{m_\mu^5 G_F^2}{192 \pi^3}. \quad (14)$$

In the model we have tree-level LFV processes mediated by the flavons which give rise to effective operators. They can be parametrized as

$$\frac{1}{\tilde{\Lambda}^2} \left\{ \eta_{LL} (\bar{e}(1 - \gamma_5) \mu \bar{e}(1 - \gamma_5) e) + \eta_{RR} (\bar{e}(1 + \gamma_5) \mu \bar{e}(1 + \gamma_5) e) + \right. \\ \left. \eta_{LR} (\bar{e}(1 - \gamma_5) \mu \bar{e}(1 + \gamma_5) e) + \eta_{RL} (\bar{e}(1 + \gamma_5) \mu \bar{e}(1 - \gamma_5) e) \right\}, \quad (15)$$

where $\tilde{\Lambda}$ is an effective scale given by

$$\frac{1}{\tilde{\Lambda}^2} = \frac{1}{4(4\lambda_1\lambda_2 - \lambda_3^2)(\epsilon_1\epsilon_2)^2 f^2} \quad (16)$$

and

$$\eta_{LL} = \left(\frac{R_{i1}^{e*} M_{eij}^{RL} L_{j2}^e}{f} \right) \left(\frac{R_{l1}^* M_{elk}^{RL} L_{k2}}{f} \right) F_{ijlk}(\lambda_1, \lambda_2, \lambda_3, \epsilon_1, \epsilon_2), \quad (17)$$

with similar expressions for η_{RR} , η_{LR} , η_{RL} . Notice that the effective scale $\tilde{\Lambda}$ is obtained summing on the exchanges of the two lighter massive flavons, $\varphi_{1,2}$, that are the only ones which give rise to tree level processes. M_e^{RL} in eq. (17) is the non diagonal charged lepton mass matrix (see the Appendix), L^e and R^e are defined in sec. (II), F_{ijlk} is a function of the potential parameters $\lambda_{i=1,2,3}$ discussed in [1] and of $\epsilon_{1,2}$ that depends on the processes multiplicities in the current basis.

From eqs. (16)–(17) we can readily compute $\Gamma_{\mu \rightarrow e^+ e^+ e^-}$ that is given by

$$\Gamma_{\mu \rightarrow 3e}^{flavoni} = \frac{m_\mu^5}{6(16\pi)^3} \left(\frac{1}{(4\lambda_1\lambda_2 - \lambda_3^2)(\epsilon_1\epsilon_2)^2 f^2} \right)^2 (|\eta_{LL}|^2 + |\eta_{RR}|^2 + |\eta_{LR}|^2 + |\eta_{RL}|^2). \quad (18)$$

By imposing the experimental bound in eq. (13) and using eq. (14) we find

$$f > 200 \text{ GeV}. \quad (19)$$

Such a rather weak bound is justified by the strong suppression of this process. This is best understood by going back to the current eigenstates. In this basis we have nine processes

that sum to give $\mu \rightarrow 3e$ in the mass eigenstates. For simplicity we consider only one of them. The interaction terms that give rise to the tree level process are

$$\lambda_{2e}\bar{e}_R^1(h^{0*}e_L^2)\left(\frac{\phi_2^\dagger\phi_1}{f^2}\right)\frac{\phi_1^0}{f} + \lambda_{1e}\bar{e}_R^1(h^{0*}e_L^1)\left(\frac{\phi_2^\dagger\phi_1}{f^2}\right)^4 + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2). \quad (20)$$

After the flavor and the electroweak spontaneous breaking the tree level effective coupling is (see Fig. (1))

$$\frac{\lambda_{2e}\lambda_{1e}}{4(4\lambda_1\lambda_2 - \lambda_3^2)(\epsilon_1\epsilon_2)^2f^2}\epsilon_1(\epsilon_1\epsilon_2)^5\frac{\langle h^{0*} \rangle^2}{f^2}(\bar{e}(1-\gamma_5)\mu)(\bar{e}(1-\gamma_5)e)(16\lambda_1 + 8\lambda_2 + 12\lambda_3), \quad (21)$$

where $(16\lambda_1 + 8\lambda_2 + 12\lambda_3)$ is the function indicated as F_{1211} in eq. (17). Processes mediated by the flavons are suppressed by powers of $\epsilon_{1,2}$ and by the ratio between the electroweak breaking scale and the flavor one.

Let us consider also a flavor changing neutral current (FCNC) process mediated by the flavons in the quark sector. FCNC processes in the quark sector with $\Delta F = 2$ are responsible of meson-antimeson oscillations. Since meson mass eigenstates are a combination of mesons in the current basis, the splitting of the masses of the mass eigenstates is related to the possible FCNC processes. This statement is general and can be applied to K^0 - \bar{K}^0 as well as B^0 - \bar{B}^0 system. Nevertheless, the best experimental data are related to the splitting of Kaon masses [7]

$$\Delta m_{LS} = (3.46 \pm 0.01) \times 10^{-12} \text{ MeV}, \quad (22)$$

and therefore we will consider only the processes with $\Delta S = 2$. Given an effective interaction $\mathcal{V} = C\mathcal{O}_{\Delta S=2}$, where C is a numerical coefficient and $\mathcal{O}_{\Delta S=2}$ the effective operator involving the quarks d and s , we have that

$$\Delta m_{LS} = 2C \frac{\text{Re} \langle K^0 | \mathcal{O}_{\Delta S=2} | \bar{K}^0 \rangle}{2m_K}. \quad (23)$$

In order to estimate the contribution of the flavons to Δm_{LS} we have to consider all the possible effective operators with $\Delta S = 2$. We have three main operators that we parametrize as follows

$$-\frac{1}{\tilde{\Lambda}^2} \left[\rho_1 \left(\bar{d}(1+\gamma_5)s \bar{d}(1-\gamma_5)s \right) + \rho_2 \left(\bar{d}(1-\gamma_5)s \bar{d}(1-\gamma_5)s \right) + \rho_3 \left(\bar{d}(1+\gamma_5)s \bar{d}(1+\gamma_5)s \right) \right], \quad (24)$$

where

$$\frac{1}{\tilde{\Lambda}^2} = \frac{2(\lambda_1 + \lambda_2 + \lambda_3)}{4(4\lambda_1\lambda_2 - \lambda_3^2)(\epsilon_1\epsilon_2)^2f^2}. \quad (25)$$

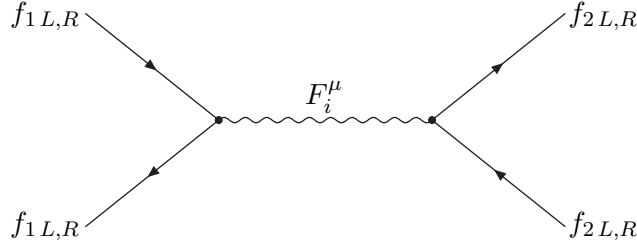


FIG. 2: Processes of annihilation and production of $f\bar{f}$ mediated by gauge flavons.

The coefficients ρ_i are given by

$$\begin{aligned}\rho_1 &= \left(\frac{\sum_{ji} (R_{j1}^{d*} M_{dji}^{RL} N_{ji} L_{i2}^d)}{f} \right) \left(\frac{\sum_{lk} (R_{l2}^{d*} M_{d lk}^{RL} N_{lk} L_{k1}^d)^*}{f} \right), \\ \rho_2 &= \left(\frac{\sum_{ji} (R_{j1}^{d*} M_{dji}^{RL} N_{ji} L_{i2}^d)}{f} \right)^2, \\ \rho_3 &= \left(\frac{\sum_{lk} (R_{l2}^{d*} M_{d lk}^{RL} N_{lk} L_{k1}^d)^*}{f} \right)^2,\end{aligned}\tag{26}$$

where M_{dji}^{RL} is the non diagonal quark mass matrices (see the Appendix) and N_{ji} is a multiplicity factor.

A comparison between eq. (22) and eq. (23) with eq. (24) indicates that we need f at least

$$f \simeq 10 \text{ TeV},\tag{27}$$

in order to satisfy the experimental bound.

However, as we shall see in the next sections, processes mediated by flavons are not the dominant ones and the limit obtained here must be increase. For this reason we will not further discuss this kind of processes, and concentrate on those mediated by the flavor gauge bosons.

IV. PROCESSES MEDIATED BY THE GAUGE FLAVONS

Processes mediated by the gauge bosons of the flavor groups are crucial in fixing the scale f . Most of the processes we discuss in the following arise from two classes of operators of the general form

$$\frac{1}{\Lambda^2} (\bar{f}_1 \Gamma^{V,A,S,P} f_1) (\bar{f}_2 \Gamma_{V,A,S,P} f_2) \quad \text{and} \quad \frac{1}{\Lambda^2} (\bar{f}_1 \Gamma^{V,A,S,P} f_2) (\bar{f}_3 \Gamma_{V,A,S,P} f_4),\tag{28}$$

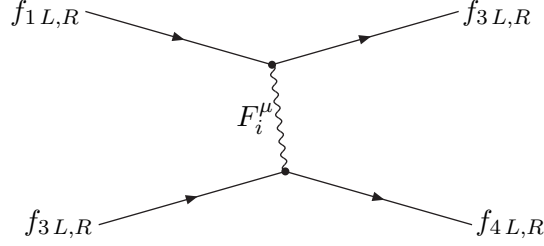


FIG. 3: Parity violation processes mediated by gauge flavons.

where in the second class of operators at least $f_2 \neq f_1$ (or $f_4 \neq f_3$). These operators arise from integrating out the gauge flavons. Notice that the longitudinal components of the gauge flavons propagators give contributions sub-leading with respect to that arising from the transverse components. The first class of operators in eq. (28) gives rise to processes of annihilation and production of fermion-antifermion couples and parity violation processes (see Fig. (2)), while the latter to flavor changing processes (see Fig. (3)). Tree level processes can give only the vectorial and the axial structure, while the scalar and pseudoscalar ones arise when we consider processes at least at one-loop. For this reason these structures are suppressed and we neglect them in the following.

A. $f\bar{f} \rightarrow f'\bar{f}'$ and parity violation

The number of four fermions operators belonging to the first class of eq. (28) which give rise to $f\bar{f} \rightarrow f'\bar{f}'$ and parity violation is very large, so we consider only those that contribute to the experimentally most constrained processes, that is, $e^+e^- \rightarrow e^+e^-$ and $e q_{L,R} \rightarrow e q_{R,L}$. They can be parametrized as

$$\begin{aligned}
& -\frac{1}{f^2} \left\{ \eta_{LL}^{ee} (\bar{e}_L \gamma_\mu e_L \bar{e}_L \gamma^\mu e_L) + \eta_{RR}^{ee} (\bar{e}_R \gamma_\mu e_R \bar{e}_R \gamma^\mu e_R) + 2\eta_{LR}^{ee} (\bar{e}_L \gamma_\mu e_L \bar{e}_R \gamma^\mu e_R) + \right. \\
& \eta_L^e \eta_L^u (\bar{e}_L \gamma_\mu e_L \bar{u}_L \gamma^\mu u_L) + \eta_R^e \eta_R^u (\bar{e}_R \gamma_\mu e_R \bar{u}_R \gamma^\mu u_R) + \eta_L^e \eta_R^u (\bar{e}_L \gamma_\mu e_L \bar{u}_R \gamma^\mu u_R) + \\
& \left. \eta_R^e \eta_L^u (\bar{e}_R \gamma_\mu e_R \bar{u}_L \gamma^\mu u_L) + (u \rightarrow d) \right\}. \tag{29}
\end{aligned}$$

The first line of eq. (29) has to be compared with the usual effective lagrangian of contact interactions [5]

$$\begin{aligned}
& \frac{g^2}{\Lambda_{LL}^2} (\pm \bar{e}_L \gamma_\mu e_L \bar{e}_L \gamma^\mu e_L) + \frac{g^2}{\Lambda_{RR}^2} (\pm \bar{e}_R \gamma_\mu e_R \bar{e}_R \gamma^\mu e_R) + \frac{g^2}{\Lambda_{LR}^2} (\pm \bar{e}_L \gamma_\mu e_L \bar{e}_R \gamma^\mu e_R) + \frac{g^2}{\Lambda_{RL}^2} (\pm \bar{e}_R \gamma_\mu e_R \bar{e}_L \gamma^\mu e_L) \\
& \tag{30}
\end{aligned}$$

where the limits on Λ_{UU} , with $U = L, R$, are usually given imposing $g^2 = 4\pi$. From [6] we have

$$\Lambda_{LL}^+ = 8.3 \text{ TeV} \text{ and } \Lambda_{LL}^- = 10.1 \text{ TeV}. \quad (31)$$

To compare these values with eq. (29), we write the η coefficients in terms of the model parameters as

$$\begin{aligned} \eta_{LL}^{ee} &= \frac{(y_{1L}^{e11})^2}{\epsilon_1^2} + \frac{(y_{2L}^{e11})^2}{\epsilon_2^2} + 2\frac{(y_{3L}^{e11})^2}{\epsilon_1^2 + \epsilon_2^2}, \\ \eta_{RR}^{ee} &= \frac{(y_{1R}^{e11})^2}{\epsilon_1^2} + \frac{(y_{2R}^{e11})^2}{\epsilon_2^2} + 2\frac{(y_{3R}^{e11})^2}{\epsilon_1^2 + \epsilon_2^2}, \\ \eta_{LR}^{ee} &= \frac{y_{1L}^{e11}y_{1R}^{e11}}{\epsilon_1^2} + \frac{y_{2L}^{e11}y_{2R}^{e11}}{\epsilon_2^2} + 2\frac{y_{3L}^{e11}y_{3R}^{e11}}{\epsilon_1^2 + \epsilon_2^2}, \end{aligned}$$

where $y_{mU}^{e\alpha\beta}$ have been defined in eq. (10). A direct comparison imposes

$$f \geq 36 \text{ TeV}, \quad (32)$$

which is two order of magnitude bigger than the value we found in the previous section for LFV.

Let us turn now to parity violation processes. Parity violation is measured in term of the weak charge Q_W and the most recent experimental values give [5]

$$\Delta Q_w = 0.44 \pm 0.44. \quad (33)$$

From the contact parameters, ΔQ_w receives the contributions [5]

$$\Delta Q_w = (-11.4 \text{ TeV}^2)(-\tilde{\eta}_{LL}^{eu} + \tilde{\eta}_{RR}^{eu} - \tilde{\eta}_{LR}^{eu} + \tilde{\eta}_{RL}^{eu}) + (-12.8 \text{ TeV}^2)(-\tilde{\eta}_{LL}^{ed} + \tilde{\eta}_{RR}^{ed} - \tilde{\eta}_{LR}^{ed} + \tilde{\eta}_{RL}^{ed}), \quad (34)$$

where

$$\tilde{\eta}_{AB}^{eq} = \frac{4\pi}{\Lambda_{AB}^{2eq}} \eta_A^e \eta_B^q. \quad (35)$$

In eq. (29) $4\pi/\Lambda_{AB}^{2eq} = -1/f^2$ and the η coefficients are given by

$$\begin{aligned} \eta_L^e &= \left(\frac{y_{1L}^{e11}}{\epsilon_1^2} + \frac{y_{2L}^{e11}}{\epsilon_2^2} \right), \\ \eta_R^e &= \left(\frac{y_{1R}^{e11}}{\epsilon_1^2} + \frac{y_{2R}^{e11}}{\epsilon_2^2} \right), \\ \eta_L^u &= y_L^{u11}, \\ \eta_R^u &= y_R^{u11}, \\ \eta_L^d &= y_L^{d11}, \\ \eta_R^d &= y_R^{d11}, \end{aligned}$$

where $y_{m_U}^{e\alpha\beta}$ and $y_U^{q\alpha\beta}$ are given in eqs. (10)–(12). A direct comparison of eq. (33) with eq. (34) gives

$$f \geq 88 \text{ TeV}. \quad (36)$$

B. Leptonic processes

The most stringent experimental limits for LFV processes comes from the processes $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$, but in the little-flavon model only $\mu \rightarrow 3e$ is present at tree level. As already discussed in sec. (III) the limit on the branching ratio for muon decay LFV is given by $\Gamma_{\mu \rightarrow 3e}/\Gamma_{\mu \rightarrow \text{all}} < 10^{-12}$.

As done in eq. (15) we parametrize the effective interactions as

$$-\frac{1}{f^2} \left(g_{LL} (\bar{e}_L \gamma_\mu \mu_L \bar{e}_L \gamma^\mu e_L) + g_{RR} (\bar{e}_R \gamma_\mu \mu_R \bar{e}_R \gamma^\mu e_R) + g_{LR} (\bar{e}_L \gamma_\mu \mu_L \bar{e}_R \gamma^\mu e_R) + g_{RL} (\bar{e}_R \gamma_\mu \mu_R \bar{e}_L \gamma^\mu e_L) \right), \quad (37)$$

where

$$\begin{aligned} g_{LL} &= \frac{y_{1L}^{e12} y_{1L}^{e11}}{\epsilon_1^2} + \frac{y_{2L}^{e12} y_{2L}^{e11}}{\epsilon_2^2} + 2 \frac{y_{3L}^{e12} y_{3L}^{e11}}{\epsilon_1^2 + \epsilon_2^2}, \\ g_{RR} &= \frac{y_{1R}^{e12} y_{1R}^{e11}}{\epsilon_1^2} + \frac{y_{2R}^{e12} y_{2R}^{e11}}{\epsilon_2^2} + 2 \frac{y_{3R}^{e12} y_{3R}^{e11}}{\epsilon_1^2 + \epsilon_2^2}, \\ g_{LR} &= \frac{y_{1L}^{e12} y_{1R}^{e11}}{\epsilon_1^2} + \frac{y_{2L}^{e12} y_{2R}^{e11}}{\epsilon_2^2} + 2 \frac{y_{3L}^{e12} y_{3R}^{e11}}{\epsilon_1^2 + \epsilon_2^2}, \\ g_{RL} &= \frac{y_{1R}^{e12} y_{1L}^{e11}}{\epsilon_1^2} + \frac{y_{2R}^{e12} y_{2L}^{e11}}{\epsilon_2^2} + 2 \frac{y_{3R}^{e12} y_{3L}^{e11}}{\epsilon_1^2 + \epsilon_2^2}. \end{aligned}$$

The rate decay for this process is then given by

$$\Gamma_{\mu \rightarrow 3e}^{gauge} = \frac{m_\mu^5}{6(16\pi)^3 f^4} (|g_{LL}|^2 + |g_{RR}|^2 + |g_{LR}|^2 + |g_{RL}|^2) \quad (38)$$

and to satisfy the experimental bound we need

$$f > 580 \text{ TeV}, \quad (39)$$

which give us the stringent bound so far.

C. K^0 - \bar{K}^0 mixing

As done in sec. (III) among all the FCNC processes with $\Delta F = 2$ in the quark sector that are responsible of meson-antimeson oscillations, we will consider only the processes

with $\Delta S = 2$ since the best experimental data are related to the splitting of Kaon masses [7] (see eq. (22)).

Also this time, in order to estimate the contribution of the gauge flavons to Δm_{LS} , we have to consider all the possible effective operators with $\Delta S = 2$. To give a more complete analysis, we will take into account also the operators arising at one-loop level. Accordingly we have six main operators that we parametrize as follows

$$\begin{aligned} & -\frac{1}{\Lambda_0^2} \left[\eta_1 \left(\bar{d} \gamma_\mu (1 - \gamma_5) s \bar{d} \gamma^\mu (1 - \gamma_5) s \right) + \eta_2 \left(\bar{d} \gamma_\mu (1 + \gamma_5) s \bar{d} \gamma^\mu (1 + \gamma_5) s \right) + \right. \\ & + \eta_3 \left(\bar{d} \gamma_\mu (1 - \gamma_5) s \bar{d} \gamma^\mu (1 + \gamma_5) s \right) - \frac{1}{\Lambda_1^2} \left[\eta_4 \left(\bar{d} (1 + \gamma_5) s \bar{d} (1 - \gamma_5) s \right) + \right. \\ & \left. \left. \eta_5 \left(\bar{d} (1 - \gamma_5) s \bar{d} (1 - \gamma_5) s \right) + \eta_6 \left(\bar{d} (1 + \gamma_5) s \bar{d} (1 + \gamma_5) s \right) \right] \right], \end{aligned} \quad (40)$$

where

$$\frac{1}{\Lambda_0^2} = \frac{1}{4f^2} \left(\frac{1}{\epsilon_1^2} + \frac{1}{\epsilon_2^2} \right) \quad \text{and} \quad \frac{1}{\Lambda_1^2} \simeq \frac{1}{(4\pi)^2} \frac{m_b^2}{4f^4} \left(\frac{1}{\epsilon_1^2} + \frac{1}{\epsilon_2^2} \right)^2, \quad (41)$$

and

$$\begin{aligned} \eta_1 &= (y_L^{d12})^2, \\ \eta_2 &= (y_R^{d12})^2, \\ \eta_3 &= y_L^{d12} y_R^{d12}, \\ \eta_4 &= \frac{1}{m_b^2} \left(\sum_{ij} R_{j1}^{d*} y_R^{dj} M_{dji}^{RL} y_L^{di} L_{i2}^d \right) \left(\sum_{nm} R_{n2}^d y_R^{dn} M_{dnm}^{RL*} y_L^{dm} L_{m1}^{d*} \right), \\ \eta_5 &= \frac{1}{m_b^2} \left(\sum_{ij} R_{j1}^{d*} y_R^{dj} M_{dji}^{RL} y_L^{di} L_{i2}^d \right)^2, \\ \eta_6 &= \frac{1}{m_b^2} \left(\sum_{nm} R_{n2}^d y_R^{dn} M_{dnm}^{RL*} y_L^{dm} L_{m1}^{d*} \right)^2, \end{aligned} \quad (42)$$

where y_U^{d12} are defined in eq. (12). eq. (42) gives the relationships between the effective operators of eq. (40) and the model parameters and charges. The last three operators proportional to $\eta_{4,5,6}$ respectively arise from one-loop box-diagrams in which gauge flavon bosons are exchanged. These one-loop effects are not the dominant ones since they only require f to be ≥ 2 TeV to satisfy the experimental limit, as one can check comparing eq. (22) and eq. (23) with eq. (42). The first three operators come from tree level processes and a comparison to the experimental limits indicates that they impose at least

$$f \simeq 4 \times 10^3 \text{ TeV}, \quad (43)$$

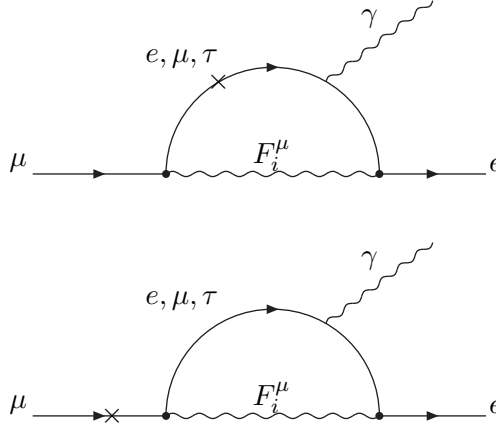


FIG. 4: Gauge flavons mediated contribution to the decay $\mu \rightarrow e\gamma$

to satisfy the bound in eq. (22). This result shifts the scale we have found in the lepton sector of more than an order of magnitude and definitively fixes the lowest scale for the breaking of the global symmetry that give rise to the little-flavons.

V. EFFECTS AT ONE LOOP

A. Rare processes

There are some rare decays that in the model occur only at one loop, but give a bound on f which is comparable to the bound obtained from the analysis of the tree-level processes.

As an example let us consider the LFV process $\mu \rightarrow e\gamma$. For the $\mu \rightarrow e\gamma$ process we have the strong limit[4]

$$\frac{\Gamma_{\mu \rightarrow e\gamma}}{\Gamma_{\mu \rightarrow \text{all}}} < 1.2 \times 10^{-11}. \quad (44)$$

We can parametrize the interaction which gives rise to the decay as

$$\left(\bar{e} i\sigma_{\nu\mu}(1 - \gamma_5) \mu \mathcal{M}^{LR} + \bar{e} i\sigma_{\nu\mu}(1 + \gamma_5) \mu \mathcal{M}^{RL} \right) F^{\nu\mu}. \quad (45)$$

In the model we have two kind of diagrams that contribute to the process $\mu \rightarrow e\gamma$ (see Fig. (4)). The second decay in Fig. (4) is present also in the Standard Model—with the charged W bosons and massive neutrinos in the loop—and gives a contribution proportional to m_μ/m_F^2 , where m_F^2 is the mass of the flavor gauge boson. On the contrary, the first is

not present in the Standard Model and is possible because the flavor gauge bosons couple also to right handed fermions. It gives a contribution proportional to $m_\alpha/m_F^2 \log(m_\alpha^2/m_F^2)$ where m_α is the mass of the fermion circulating in the loop. For this reason the dominant contribution comes from the τ exchange. For this process we have

$$\begin{aligned}\mathcal{M}^{RL} &= \frac{e}{2\pi^2} \frac{m_\tau}{f^2} \left(\log \frac{m_\tau^2}{f^2} \right) Y^{RL}, \\ \mathcal{M}^{LR} &= \frac{e}{2\pi^2} \frac{m_\tau}{f^2} \left(\log \frac{m_\tau^2}{f^2} \right) Y^{LR},\end{aligned}\tag{46}$$

where e is the electric charge and

$$\begin{aligned}Y^{RL} &= \frac{y_{1L}^{e32} y_{1R}^{e13}}{\epsilon_1^2} + \frac{y_{2L}^{e32} y_{2R}^{e13}}{\epsilon_2^2} + 2 \frac{y_{3L}^{e32} y_{3R}^{e13}}{\epsilon_1^2 + \epsilon_2^2}, \\ Y^{LR} &= \frac{y_{1R}^{e32} y_{1L}^{e13}}{\epsilon_1^2} + \frac{y_{2R}^{e32} y_{2L}^{e13}}{\epsilon_2^2} + 2 \frac{y_{3R}^{e32} y_{3L}^{e13}}{\epsilon_1^2 + \epsilon_2^2}.\end{aligned}\tag{47}$$

The rate decay for this process is then given by

$$\Gamma_{\mu \rightarrow e\gamma} = \frac{3\alpha}{8\pi^4} \frac{m_\mu^3 m_\tau^2}{f^4} \left(\log \frac{m_\tau^2}{f^2} \right)^2 (Y^{RL^2} + Y^{LR^2})\tag{48}$$

In order to satisfy the experimental bound of eq. (44) we need

$$f \simeq 4 \times 10^3 \text{ TeV}.\tag{49}$$

which is of the same order of the value obtained in sec. (IV C).

The process corresponding to the LFV process $\mu \rightarrow e\gamma$ in the quark sector is the FCNC process $b \rightarrow s\gamma$. For the $b \rightarrow s\gamma$ process we have the limit [8]

$$\frac{\Gamma_{b \rightarrow s\gamma}}{\Gamma_{b \rightarrow \text{all}}} < (3.3 \pm 0.4) \times 10^{-4}.\tag{50}$$

The Standard Model effective interaction that is responsible of this process is parametrized as [9]

$$\frac{-4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \frac{em_b}{16\pi^2} C_7(m_W) \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R,\tag{51}$$

where $C_7(m_W)$ is the Wilson coefficient and is a function of $m_t(m_W)$ and m_W as reported in [9]. In the following we will neglect of the renormalization effects and we will compare the effective interaction which gives rise to the decay $b \rightarrow s\gamma$ in our model with the one

loop electroweak operator of eq. (51). Analogously to what done for the process $\mu \rightarrow e\gamma$, we parametrize the interaction responsible of the decay $b \rightarrow s\gamma$ as

$$\left(\bar{s} i\sigma_{\nu\mu}(1 - \gamma_5) b \mathcal{N}^{LR} + \bar{s} i\sigma_{\nu\mu}(1 + \gamma_5) b \mathcal{N}^{RL}\right) F^{\nu\mu}. \quad (52)$$

All the considerations done for the process $\mu \rightarrow e\gamma$ may be applied in this context and for this reason the dominant contribution to the process $b \rightarrow s\gamma$ comes from the loops in which a quark b is exchanged. For the process we are considering we have

$$\begin{aligned} \mathcal{N}^{RL} &= \frac{e}{2\pi^2} \frac{m_b}{f^2} \left(\log \frac{m_b^2}{f^2} \right) X^{RL}, \\ \mathcal{N}^{LR} &= \frac{e}{2\pi^2} \frac{m_b}{f^2} \left(\log \frac{m_b^2}{f^2} \right) X^{LR}, \end{aligned} \quad (53)$$

where e is the electric charge and

$$\begin{aligned} X^{RL} &= y_L^{d33} y_R^{d23} \left(\frac{1}{\epsilon_1^2} + \frac{1}{\epsilon_2^2} \right), \\ X^{LR} &= y_R^{d33} y_L^{d23} \left(\frac{1}{\epsilon_1^2} + \frac{1}{\epsilon_2^2} \right). \end{aligned} \quad (54)$$

A comparison between eq. (51) and eq. (52) indicates that we need

$$f \simeq 31 \text{ TeV}, \quad (55)$$

in order to have the two contribution of the same order.

B. Muon anomalous magnetic moment

In sec. (V A) we have seen how one loop processes give a bound on f comparable to that one obtained by tree level processes. We may ask ourselves what is the limit on f we obtain if we consider the one loop contribution to the anomalous magnetic moment of the muon. The uncertainty between the experimental data and the theoretical computation for the anomalous magnetic moment of the muon $a_\mu = (g_\mu - 2)/2$ is [10]

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (27 \pm 14) \times 10^{-10}. \quad (56)$$

In order to obtain a limit for f from eq. (56) we have to consider the effective interaction that is proportional to a_μ . The interaction coincides to that of eq. (45) that is responsible

of the rare decay $\mu \rightarrow e\gamma$ once we substitute the outgoing electron with an outgoing muon. Therefore it is parametrized as

$$\left(\bar{\mu} i\sigma_{\nu\mu}(1 - \gamma_5) \mu \tilde{\mathcal{M}}^{LR} + \bar{\mu} i\sigma_{\nu\mu}(1 + \gamma_5) \mu \tilde{\mathcal{M}}^{RL}\right) F^{\nu\mu}. \quad (57)$$

As in sec. (V A) the dominant contribution comes from τ exchanging and for this reason we have

$$\begin{aligned} \tilde{\mathcal{M}}^{RL} &= \frac{e}{2\pi^2} \frac{m_\tau}{f^2} \left(\log \frac{m_\tau^2}{f^2} \right) \tilde{Y}^{RL}, \\ \tilde{\mathcal{M}}^{LR} &= \frac{e}{2\pi^2} \frac{m_\tau}{f^2} \left(\log \frac{m_\tau^2}{f^2} \right) \tilde{Y}^{LR}, \end{aligned} \quad (58)$$

where e is the electric charge and

$$\begin{aligned} \tilde{Y}^{RL} &= \frac{y_{1L}^{e32} y_{1R}^{e23}}{\epsilon_1^2} + \frac{y_{2L}^{e32} y_{2R}^{e23}}{\epsilon_2^2} + 2 \frac{y_{3L}^{e32} y_{3R}^{e23}}{\epsilon_1^2 + \epsilon_2^2}, \\ \tilde{Y}^{LR} &= \frac{y_{1R}^{e32} y_{1L}^{e23}}{\epsilon_1^2} + \frac{y_{2R}^{e32} y_{2L}^{e23}}{\epsilon_2^2} + 2 \frac{y_{3R}^{e32} y_{3L}^{e23}}{\epsilon_1^2 + \epsilon_2^2}. \end{aligned} \quad (59)$$

From eq. (57) we have

$$a_\mu = \frac{m_\mu (\tilde{\mathcal{M}}^{RL} + \tilde{\mathcal{M}}^{LR})}{e}. \quad (60)$$

A direct comparison between eq. (56) and eq. (60) gives

$$f \simeq 28 \text{ TeV}, \quad (61)$$

that does not change the previous results obtained in sec. (IV C).

VI. STABILIZATION OF THE ELECTROWEAK SCALE

From the little-flavon model point of view the stabilization of the electroweak scale must be thought as obtained by means of a little-Higgs mechanism acting on the radiative corrections to the Higgs mass. However, since the scale of our model turns out to be quite high with respect to that electroweak, we have to worry about the corrections to the Higgs mass coming from the fermionic sector and its interaction with the flavons. The potentially dangerous contributions come from one and more loop diagrams which are quadratic divergent and in which the cut-off should be taken at $\Lambda \simeq 5 \times 10^4 \text{ TeV}$. Usually little-Higgs models

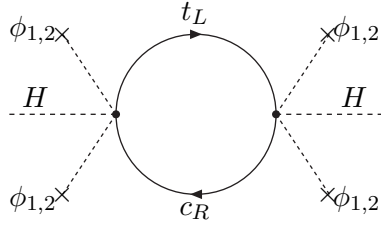


FIG. 5: Potentially dangerous fermion one-loop correction to the Higgs mass.

protect the Higgs mass from the quadratic divergences arising from one-loop corrections up to a scale $\Lambda_H \simeq 10$ TeV [2] and $f_H \simeq \Lambda_H/4\pi \simeq 1$ TeV is the scale at which the Higgs arises as a pseudo-Goldstone boson. It is possible to protect the Higgs mass even from the quadratic corrections arising from fermions two-loops if the approximate global symmetries of the little-Higgs model are enlarged. The cut-off of such a model is then shifted at $\Lambda_H \simeq 100$ TeV. In [11] an example of how this mechanism works is given.

The scale of the little-flavon model is so large that it forces us to protect the Higgs mass up to four-loops quadratic divergences and this is done by further enlarging the global symmetries of the little-Higgs model. Since the choice of the little-Higgs model is independent of the little-flavon model, we shall give an example of how this mechanism is applied to the Littlest Higgs model that is based on an $SU(5)$ global symmetry (first reference in Ref.[2]).

For each Standard Model family, both for quarks and the leptons, U_i and U_i^c , we add four quintuplet, X_i , X_i^c , Y_i , Y_i^c , in order to have an approximate global symmetry $[SU(5)]^5$. Only X_i^c and X_i are charged under the flavor gauge group $SU(2)_F \times U(1)_F$ and their charges are that of the corresponding Standard Model fermions U_i and U_i^c . The Yukawa lagrangian is then given only by terms that leave invariant at least one of the five $SU(5)$ and takes the following expression

$$\mathcal{L}_Y = \lambda_1 f_H A_{ij} X_i \Sigma_H X_j^c + \lambda_2 f_H X_j^c Y_j + \lambda_3 f_H Y_j Y_j^c + \lambda_4 f_H Y_j^c U_j + \lambda_5 f_H X_i U_i^c, \quad (62)$$

where with A_{ij} we indicate the flavons which couple to X_i and X_j^c and with Σ_H the non linear representations of the little-Higgs model pseudo-Goldstone bosons. In this way the Higgs mass receives a quadratic divergence only from the diagrams in which all the $SU(5)$ approximate global symmetries are broken and this happens only from five loops on. The light fermions, \tilde{q}_j and \tilde{u}_i^c , are now given by appropriate combinations of the fields appearing in eq. (62) and the effective coupling between light fermions, Higgs and flavons is then given

by

$$\mathcal{L}_Y = \frac{\lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5}{\sqrt{\lambda_1^2 + \lambda_2^2} \sqrt{\lambda_1^2 + \lambda_5^2} \sqrt{\lambda_3^2 + \lambda_4^2} \sqrt{\lambda_2^2 + \lambda_3^2}} A_{ij} \tilde{u}_i^c \tilde{q}_j h. \quad (63)$$

The expression for \mathcal{L}_Y reported in the Appendix must therefore be read as effective terms, the explicit form of which is like that in eq. (63).

VII. CONCLUSION

While the results obtained in [1] are independent of the scale f and the masses and mixing matrices are determined only by the value of ϵ_1 and ϵ_2 , a detailed analysis of the flavor changing processes induced by the new particles introduced by the little-flavon model shows that this model has to live at least at a scale $\Lambda = 4\pi f \simeq 5 \times 10^4$ TeV.

This result has two consequences for the little-flavon model. On the one hand, the determination of a bound on the scale f leads to a specific prediction for the scale for the see-saw mechanism that in the model is used to give mass to the neutrinos. The value found of 10^4 TeV allows to have the couplings of the Dirac neutrino mass term to be of the same order of the charged lepton ones. As consequence these couplings are of order 10^{-2} , two order of magnitude below those of the previous rough estimate [1].

On the other hand, the scale of the model turns out to be quite high with respect to that of the electroweak symmetry breaking, thus making rather challenging a hypothetical unification of the little-flavon and the little-Higgs models in a single gauge and flavor symmetry scenario. However, the high scale of the little flavon model is not dangerous for the electroweak scale since it is possible to keep the latter stable—in the fermion sector— up to the scale $\Lambda \simeq 5 \times 10^4$ TeV, as argued in sec. (VI).

VIII. ACKNOWLEDGMENT

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APPENDIX

The Yukawa lagrangian \mathcal{L}_Y is rather cumbersome. Here we give its expression as a function of $\Sigma = \exp(i\Pi/f)\Sigma_0$ and the charged lepton and quark fields as discussed in [1]. In this context we shall neglect the neutrino term since neutrinos do not enter in any process we have considered. We also give the charge assignments for all the fields that enter in \mathcal{L}_Y (see Tab. (I)).

$$\mathcal{L}_Y = \mathcal{L}_u + \mathcal{L}_d + \mathcal{L}_e, \quad (64)$$

where

$$\begin{aligned} -\mathcal{L}_u = & \lambda_{31}\overline{t}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^3\tilde{H}^\dagger Q_{1L} + \lambda_{32}\overline{t}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^2\tilde{H}^\dagger Q_{2L} \\ & + \overline{t}_R(\lambda_{33} + \lambda'_{33}\Sigma_{\alpha-16}\Sigma_{32+\alpha} + \lambda''_{33}\Sigma_{\alpha-13}\Sigma_{62+\alpha})\tilde{H}^\dagger Q_{3L} \\ & + \lambda_{21}\overline{c}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^4\tilde{H}^\dagger Q_{1L} + \lambda_{22}\overline{c}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^3\tilde{H}^\dagger Q_{2L} \\ & + \lambda_{23}\overline{c}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})\tilde{H}^\dagger Q_{3L} \\ & + \lambda_{11}\overline{u}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^6\tilde{H}^\dagger Q_{1L} + \lambda_{12}\overline{u}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^5\tilde{H}^\dagger Q_{2L} \\ & + \lambda_{13}\overline{u}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^3\tilde{H}^\dagger Q_{3L} + H.c. \end{aligned}$$

$$\begin{aligned} -\mathcal{L}_d = & \tilde{\lambda}_{31}\overline{b}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^5H^\dagger Q_{1L} + \tilde{\lambda}_{32}\overline{b}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^4H^\dagger Q_{2L} \\ & + \tilde{\lambda}_{33}\overline{b}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^2H^\dagger Q_{3L} \\ & + \tilde{\lambda}_{21}\overline{s}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^5H^\dagger Q_{1L} + \tilde{\lambda}_{22}\overline{s}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^4H^\dagger Q_{2L} \\ & + \tilde{\lambda}_{23}\overline{s}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^2H^\dagger Q_{3L} \\ & + \tilde{\lambda}_{11}\overline{d}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^7H^\dagger Q_{1L} + \tilde{\lambda}_{12}\overline{d}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^6H^\dagger Q_{2L} \\ & + \tilde{\lambda}_{13}\overline{d}_R(\Sigma_{\alpha-13}\Sigma_{32+\alpha})^4H^\dagger Q_{3L} + H.c. \end{aligned}$$

$$\begin{aligned} \mathcal{L}_e = & \overline{e}_R \left[\lambda_{1e} (\Sigma_{\alpha-16}\Sigma_{62+\alpha})^{(-Y_{1L}+Y_{1R})} (H^\dagger l_{1L}) \right. \\ & \left. + i (\lambda_{3e} \Sigma_{62+\alpha} + \lambda_{2e} \epsilon_{\alpha\beta}\Sigma_{\beta-16})(\Sigma_{\delta-16}\Sigma_{62+\delta})^{(Y_{1R}-1)} (H^\dagger l_{\alpha L}) \right] \\ & + \overline{E}_{\alpha R} \left[i (\lambda'_{1E} \Sigma_{62+\alpha} + \lambda_{1E} \epsilon_{\alpha\beta}\Sigma_{\beta-16}) (\Sigma_{\delta-16}\Sigma_{62+\delta})^{-Y_{1L}} \right] (H^\dagger l_{1L}) \\ & + \overline{E}_{\alpha R} \left[\delta_{\alpha\beta} (-\lambda_{2E} + \lambda'_{2E} \Sigma_{\gamma-16}\Sigma_{32+\gamma} + \lambda''_{2E} \Sigma_{\gamma-13}\Sigma_{62+\gamma}) \right. \\ & \left. + (\lambda_{3E} \Sigma_{\alpha-16}\Sigma_{32+\beta} + \lambda'_{3E} \epsilon_{\alpha\delta}\epsilon_{\beta\gamma} \Sigma_{62+\delta}\Sigma_{\gamma-13} + (3 \leftrightarrow 6)) \right. \\ & \left. + (\lambda_{4E} \Sigma_{\alpha-16}\Sigma_{\gamma-13}\epsilon_{\beta\gamma} + \lambda'_{4E} \epsilon_{\alpha\delta}\Sigma_{62+\delta}\Sigma_{3,2+\alpha} + (3 \leftrightarrow 6)) \right] (H^\dagger l_{\beta L}) + H.c. \quad (65) \end{aligned}$$

TABLE I: Charges of fermion and flavon fields ($\alpha = 2, 3$) under the horizontal flavor groups $SU(2)_F$ and $U(1)_F$. l_{fL} and Q_{iL} stands for the electroweak left-handed doublets. q is an arbitrary not determined charge and being universal in the quark sector does not affect any process.

	$U(1)_F$	$SU(2)_F$
l_{eL}	-2	1
e_R	2	1
$L_L = (l_\mu, l_\tau)_L$	$1/2$	2
$E_R = (\mu, \tau)_R$	$1/2$	2
ν_{1R}	1	1
ν_{2R}	-1	1
ν_{3R}	0	1
Q_{1L}	$q + 3$	1
Q_{2L}	$q + 2$	1
Q_{3L}	q	1
u_R	$q - 3$	1
c_R	$q - 1$	1
t_R	q	1
d_R	$q - 4$	1
s_R	$q - 2$	1
b_R	$q - 2$	1
$\Sigma_{\alpha-16} = (-i/f \phi_1 + \dots)_{\alpha-1}$	$1/2$	2
$\Sigma_{\alpha-13} = (+i/f \phi_2 + \dots)_{\alpha-1}$	$-1/2$	2
$\Sigma_{32+\alpha} = (-i/f \phi_1^* + \dots)_{\alpha-1}$	$-1/2$	2^*
$\Sigma_{62+\alpha} = (-i/f \phi_2^* + \dots)_{\alpha-1}$	$1/2$	2^*

The charged lepton and quark mass matrices in the current basis are given by

$$M_e^{RL} = \langle h_0 \rangle \begin{pmatrix} \lambda_{1e} \varepsilon_1^4 \varepsilon_2^4 & \lambda_{2e} \varepsilon_1^2 \varepsilon_2^2 & \lambda_{3e} \varepsilon_1 \varepsilon_2^2 \\ \lambda_{1E} \varepsilon_1^2 \varepsilon_2^3 & \lambda_{2E} & (\lambda'_{14E} + \lambda'_{24E}) \varepsilon_1 \varepsilon_2 \\ \lambda'_{1E} \varepsilon_1^3 \varepsilon_2^2 & -(\lambda_{14E} + \lambda_{24E}) \varepsilon_1 \varepsilon_2 & \lambda_{2E} \end{pmatrix}, \quad (66)$$

where the notation follows that of eq. (39) in ref. [1], and

$$M_u^{RL} = \langle h_0 \rangle \begin{pmatrix} \lambda_{11} k^6 & \lambda_{12} k^5 & \lambda_{13} k^3 \\ \lambda_{21} k^4 & \lambda_{22} k^3 & \lambda_{23} k \\ \lambda_{31} k^3 & \lambda_{32} k^2 & \lambda_{33} \end{pmatrix} \text{ and } M_d^{RL} = \langle h_0 \rangle k^2 \begin{pmatrix} \tilde{\lambda}_{11} k^5 & \tilde{\lambda}_{12} k^4 & \tilde{\lambda}_{13} k^2 \\ \tilde{\lambda}_{21} k^3 & \tilde{\lambda}_{22} k^2 & \tilde{\lambda}_{23} \\ \tilde{\lambda}_{31} k^3 & \tilde{\lambda}_{32} k^2 & \tilde{\lambda}_{33} \end{pmatrix}, \quad (67)$$

where $k = \epsilon_1 \epsilon_2$.